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Information Loss in Local Dissipation Environments

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Abstract The sensitivity of entanglement to the thermal and squeezed reservoirs' parameters is investigated regarding entanglement decay and what is called sudden-death of entanglement, for a system of two qubit pairs. The dynamics of information is investigated by means of the information disturbance and exchange information. We show that for squeezed reservoir, we can keep both of the entanglement and information survival for a long time. However the sudden death of information is depicted clearly when the entangled qubit system interacts with thermal reservoir. By controlling the environmental parameters, one can reduces the lose of the encrypted information.

Keywords Entanglement · Information · Disturbance · Exchange information

1 Introduction

Entangled qubits are one of the most promising candidates for quantum communication and computation. There are many interesting applications based on these entangled systems. Among these applications, dense coding [1], quantum teleportation [2], quantum cryptography [3] and etc. These entangled systems can not be isolated from their surrounding environments. So, investigating and quantifying the amount of entanglement contained in entangled system interacting with open systems is very important in the context of quantum information [4]. As an example, Yu and Eberly [5, 6], have investigated the dynamics of entanglement for entangled qubit pairs undergoing various modes of decoherence. They showed that the dynamics of entanglement between two qubits system interacting independently with classical or quantum noise, displays two different types of behavior; the phenomena of entanglement decay and entanglement sudden death, ESD [6–9]. In some systems, the

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ESD appears whenever the system is open or closed [10]. The effect of the local squeezed reservoir on initially entangled two qubits system is investigated, where it is shown that the squeezing causes different behavior of entanglement decay on different time scales [11]. Recently in [12], it has been shown that the ESD always exists with thermal and squeezed reservoirs, where the authors presented explicit expression for the ESD for some entangled states. Also, it has been shown, that the ESD and entanglement decay phenomena appear for qubit system passed through Bloch channel [13–15]. The ESD under the effect of the individual environments has been experimentally seen [16].

The purpose of this paper is to continue investigating questions of this sort: How much are the entanglement and the information of entangled two qubits state degrade when it passes through a thermal or squeezed reservoir.

The paper is organized as follows: In Sect. 2, we present the model and its solution. The dynamics of entanglement is investigated in Sect. 3, where we consider two classes of entangled states as an initial states, maximum and partial entangled states. The amount of disturbance and exchange information between the entangled state and the local reservoirs are discussed in Sect. 4. Finally a conclusion is given in Sect. 5.

2 The Model and Its Solution

Assume that we have a source generates entangled qubit pairs. One qubit is sent to Alice and the other to Bob. The general two qubits state is given by

$$\rho_{ab}(0) = \frac{1}{4}(1 + \vec{s} \cdot \sigma_1^{\downarrow} + \vec{t} \cdot \sigma_2^{\downarrow} + \vec{\sigma}_1 \cdot \overleftrightarrow{C} \cdot \sigma_2^{\downarrow}), \tag{1}$$

where the vectors $\vec{s}$ and $\vec{t}$ are the Bloch vectors for Alice and Bob's qubits respectively, $\overleftrightarrow{C}$ is a 3×3 matrix represents the cross dyadic and $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the spin Pauli vectors [17, 18]. Now assume that each qubit interacts individually with its squeezed vacuum environment [11]. Within Markov approximation, the master equation in the Schrödinger form is,

$$\frac{\partial}{\partial t}\rho_{ab} = L_a(\rho_{ab}) + L_b(\rho_{ab}), \tag{2}$$

with,

$$\begin{aligned} L_i(\rho_{ab}) = &-\frac{\Gamma_i}{2}(1+\mathcal{N}_i)(\sigma_i^+\sigma_i^-\rho_{ab} - 2\sigma_i^-\rho_{12}\sigma_i^+ + \rho_{ab}\sigma_i^+\sigma_i^-) \\ &-\frac{\Gamma_i}{2}\mathcal{N}_i(\sigma_i^-\sigma_i^+\rho_{ab} - 2\sigma_i^+\rho_{12}\sigma_i^- + \rho_{ab}\sigma_i^-\sigma_i^+) \\ &-\frac{\Gamma_i}{2}\mathcal{M}_i(\sigma_i^+\sigma_i^+\rho_{ab} - 2\sigma_i^+\rho_{12}\sigma_i^+ + \rho_{ab}\sigma_i^+\sigma_i^+) \\ &-\frac{\Gamma_i}{2}\mathcal{M}_i^*(\sigma_i^-\sigma_i^-\rho_{ab} - 2\sigma_i^-\rho_{12}\sigma_i^- + \rho_{ab}\sigma_i^-\sigma_i^-), \end{aligned} \tag{3}$$

where $i = 1, 2$ refers to the first (Alice's qubit) and the second for (Bob's qubit), Γ_i is the atomic spontaneous emission rate for local squeezed field. The parameter $\mathcal{M} = |\mathcal{M}|e^{i\theta}$, describes the strength of the two photons correlation, where $|\mathcal{M}_i| \le \sqrt{\mathcal{N}_i(1+\mathcal{N}_i)}$. Finally, $\sigma_i^{\pm} = \sigma_{ix} \pm i\sigma_{iy}$.

To investigate the dynamical behavior of the initial entangled state $\rho_{ab}(0)$, we solve the Schrödinger equation (2). In this context, we use the Kraus representation described in [11]. The time-evolution of the input state (1) is given by

$$\rho_{ab}(t) = \sum_{j}^{4} \kappa_j^a \otimes \kappa_j^b \rho_{ab}(0) \kappa_j^{a\dagger} \kappa_j^{b\dagger}. \tag{4}$$

For our analysis, we describe the Kraus operators in the computational basis $|0\rangle$ and $|1\rangle$ as,

$$\begin{aligned} \kappa_1^i &= \alpha_1^i |0\rangle\langle 1| + \beta_1^i |1\rangle\langle 1|, \qquad \kappa_2^i = \beta_2^i |1\rangle\langle 1|, \\ \kappa_3^i &= \alpha_3^i |0\rangle\langle 1| + \beta_3^i |1\rangle\langle 0|, \qquad \kappa_4^i = \alpha_4^i |1\rangle\langle 0|, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \alpha_1^i &= e^{-\frac{\zeta_i t}{2}} \sqrt{\cosh \zeta_i t + \frac{\Gamma_i}{2\zeta_i} \sinh \zeta_i t}, \qquad \beta_1^i = e^{-\frac{\zeta_i t}{2}} \frac{\cosh \eta_i t}{\alpha_1^i}, \\ \beta_2^i &= e^{-\frac{\zeta_i t}{2}} \sqrt{\frac{[1 - (\frac{\Gamma_i}{2\zeta_i})^2] \sinh^2 \zeta_i t - \sinh^2 \eta_i t}{\cosh \zeta_i t + \frac{\Gamma_i}{2\zeta_i} \sinh \zeta_i t}}, \\ \alpha_3^i &= e^{-\frac{\zeta_i t}{2}} \frac{\sinh(\eta_i t)}{\sqrt{(1 + \frac{\Gamma_i}{2\zeta_i}) \sinh(\eta_i t)}}, \qquad \beta_3^i = e^{-\frac{\zeta_i t}{2}} \sqrt{\left(1 + \frac{\Gamma_i}{2\eta_i}\right) \sinh(\eta_i t)}\, e^{-i\theta}, \\ \alpha_4^i &= e^{-\frac{\zeta_i t}{2}} \sqrt{\frac{[1 - (\frac{\Gamma_i}{2\zeta_i})^2] \sinh^2(\zeta_i t) - \sinh^2(\eta_i t)}{(1 + \frac{\Gamma_i}{2\eta_i}) \sinh(\eta_i t)}}, \end{aligned} \tag{6}$$

$\zeta_i = \frac{\Gamma_i}{2}(2\mathcal{N}_i + 1)$ and $\eta_i = \Gamma_i |\mathcal{M}_i|,\ i = 1, 2$.

To show our idea, let us consider a class of entangled states with zero Bloch vectors, ($\vec{s} = \vec{t} = 0$). This simplification leads to what is called a generalized Werner state [19, 20],

$$\rho(0) = \frac{1}{4}(1 + c_1 \sigma_{1x}\sigma_{2x} + c_2 \sigma_{1y}\sigma_{2y} + c_3 \sigma_{1z}\sigma_{2z}). \tag{7}$$

By means of Bell states $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, one can rewrite the initial state (7) as,

$$\begin{aligned} \rho(0) = {} & \frac{1 + c_1 + c_2 + c_3}{4} |\phi^+\rangle\langle\phi^+| + \frac{1 - c_1 + c_2 + c_3}{4} |\phi^-\rangle\langle\phi^-| \\ & + \frac{1 - c_1 - c_2 - c_3}{4} |\psi^-\rangle\langle\psi^-| + \frac{1 + c_1 + c_2 - c_3}{4} |\psi^+\rangle\langle\psi^+|. \end{aligned} \tag{8}$$

From this class of states, we can get the singlet state $|\psi^-\rangle\langle\psi^-|$ if we set $c_1 = c_2 = c_3 = -1$, and if $c_1 = c_2 = c_3 = 1$, one gets $|\phi^+\rangle\langle\phi^+|$ and so on. Also, if $c_1 = c_2 = c_3 = -x$, one gets Werner state [19],

$$\rho_w(0) = \frac{3x + 1}{4} |\psi^-\rangle\langle\psi^-| + \frac{1 - x}{4} \big(|\phi^+\rangle\langle\phi^+| + |\phi^-\rangle\langle\phi^-| + |\psi^+\rangle\langle\psi^+|\big). \tag{9}$$

Now, let us assume that a source supplies us with entangled qubit states of form (8). The qubits leave each other and then interact with their local reservoirs. The time evolvement of the density operator (8) is,

$$\begin{aligned}\rho(t) = e^{-i\Gamma t}\Big[&\frac{1+c_3}{8}\{(s_1+s_4)(|\phi^+\rangle\langle\phi^+|+|\phi^-\rangle\langle\phi^-|)+(s_1-s_4)(|\phi^+\rangle\langle\phi^-|+|\phi^-\rangle\langle\phi^+|)\}\\ &+\frac{c_1-c_2}{8}\{(s_2+s_3)(|\phi^+\rangle\langle\phi^+|-|\phi^-\rangle\langle\phi^-|)+(s_2-s_3)(|\phi^-\rangle\langle\phi^+|-|\phi^+\rangle\langle\phi^-|)\}\\ &+\frac{1-c_3}{8}\{(s_5+s_8)(|\psi^+\rangle\langle\psi^+|+|\psi^-\rangle\langle\psi^-|)+(s_5-s_8)(|\psi^+\rangle\langle\psi^-|+|\psi^-\rangle\langle\psi^+|)\}\\ &+\frac{c_1+c_2}{8}\{(s_6+s_7)(|\psi^+\rangle\langle\psi^+|-|\psi^-\rangle\langle\psi^-|)+(s_6-s_7)(|\psi^-\rangle\langle\psi^+|-|\psi^+\rangle\langle\psi^-|)\}\Big],\end{aligned} \tag{10}$$

where,

$$\begin{aligned}&s_1=\sum_{i=1,3}|\alpha_i^a|^2|\alpha_i^b|^2, \qquad s_2=\sum_{i=1,3}\alpha_i^a\alpha_i^b\beta_i^{*a}\beta_i^{*b}, \qquad s_3=\sum_{i=1,3}\beta_i^a\beta_i^b\alpha_i^{*a}\alpha_i^{*b},\\ &s_4=\sum_{i=2,4}|\alpha_i^a|^2|\alpha_i^b|^2+\sum_{i=1,3}|\beta_i^a|^2|\beta_i^b|^2, \qquad s_5=\sum_{i=1,3}|\alpha_i^a|^2|\beta_i^b|^2,\\ &s_6=\sum_{i=1,3}\alpha_i^a\beta_i^b\beta_i^{*a}\alpha_i^{*b}, \qquad s_7=\sum_{i=1,3}\beta_i^a\alpha_i^b\alpha_i^{*a}\beta_i^{*b}, \qquad s_8=\sum_{i=1,3}|\beta_i^a|^2|\alpha_i^b|^2,\\ &\Gamma=\Gamma_1+\Gamma_2.\end{aligned} \tag{11}$$

3 Entanglement Dynamics

In this section, we investigate the robustness of the entangled state when each qubit interacts with its own reservoir individually. In our *first example*, we assume that the source supplies the users Alice and Bob with maximum entangled state say, $|\phi^+\rangle\langle\phi^+|$ with $c_1=c_2=c_3=1$ or partially entangled state with $c_1=c_2=c_3=0.85$. Unfortunately, each qubit forced to passes through individual reservoir for some time. During this time there is non desirable interactions between the qubits and the reservoirs. These interactions cause deteriorate of the amount entanglement contained in the entangled state and consequently the efficiency of performing quantum information tasks decreases. In this treatment, we consider the local reservoirs to be thermal or squeezed.

To quantify the degree of entanglement we use the negativity as a measure of entanglement [21]. It satisfies all the criteria needed in the measure and it has been proved that the negativity an entanglement monotone and therefore it is a good measure of entanglement [22, 23]. For mixed states, it gives the same results obtained from the other measures as concurrence [24]. Also, the negativity and the relative entropy of entanglement and lead to upper bounds on the entanglement [25]. The negativity is given in terms of the eigenvalues of the partial transpose of the density operator [21],

$$DoE=\sum_i|\lambda_i|-1, \tag{12}$$

where λ_i are the eigenvalues of the partial transpose of the output density operator (10).

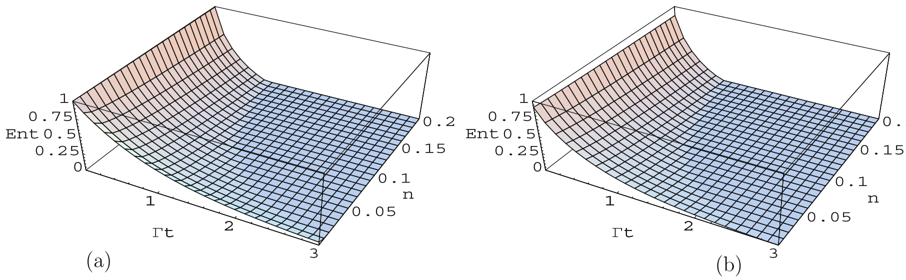

Fig. 1 The effect of the mean photon numbers $\mathcal{N}_1 = \mathcal{N}_2 = n$ on the degree of entanglement for a system is initially prepared in the (**a**) maximum entangled state $|\phi^+\rangle\langle\phi^+|$ i.e. $c_1 = c_2 = c_3 = 1$ and (**b**) partial entangled state (see (9)) with $x = 0.85$

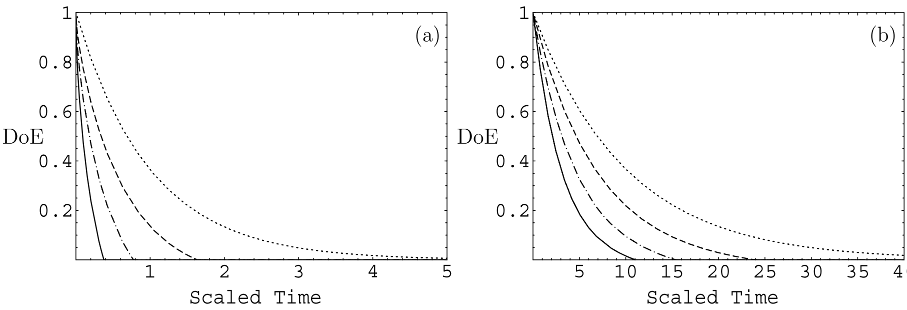

Fig. 2 The *dot*, *dash*, *dash-dot* and *solid curves* represent the degree of entanglement for a system is initially prepared in maximum entangled state (**a**) inside a thermal reservoir $\mathcal{N}_1 = \mathcal{N}_2 = n = 0.00001, 0.05, 0.2, 0.6$ and (**b**) inside a squeezed reservoir, with $\mathcal{M}_1 = \mathcal{M}_2 = 0.05, 0.2, 0.4, 0.6$ and $n = 0.2$

Figure 1, shows the dynamics of entanglement against the normalized time Γt, where the two qubits pass through a thermal reservoir, i.e. $\mathcal{M}_1 = \mathcal{M}_2 = 0$ and the two reservoirs have the same number of photons, $\mathcal{N}_1 = \mathcal{N}_2 = n$. In Fig. 1(a), the case where the source supplies the partners Alice and Bob with maximum entangled state is considered. It is clear that for small values of n, the entanglement decays asymptotically and the time of the sudden death is delayed. For large values of n the decay of entanglement is hastened and the time of entanglement sudden death becomes shorter.

In Fig. 1(b), the amount of survival amount of entanglement contained in a density operator initially prepared in a partially entangled state is quantified. It is clear that, the entanglement decays faster and the time of the entanglement sudden death is shorter. These figures show that the entanglement decay and the sudden death of entanglement are sensitive to the initial entangled state. So, by controlling n and Γ one can prolong the time of lived entanglement and delayed the time of the sudden death.

In Fig. 2(a), the dynamics of entanglement at some specific values of the mean photon number, n is investigated, where the qubits interact with their local thermal reservoir. For small value of ($n = 0.00001$), the phenomena of long-lived entanglement is seen, where the entanglement decreases asymptotically. On the other hand, as n increases, the time at which the entanglement vanishes decrease. The entanglement sudden death phenomena appears for much larger values of the mean photon number, $n = 6$. The behavior of the entanglement when the qubits interact with squeezed reservoir is depicted in Fig. 2(a). In this case the entanglement decays smoothly and the long lived entanglement is observed for small values of the squeezed parameter. Also, the vanishing time of entanglement is much larger than that has been shown in Fig. 2(a).

Finally, one can say that, the phenomena of sudden death and decay of entanglement not only depends on the field's parameter but also on the initial state setting. These results are coincide with that obtained in [26, 27], where the entanglement of two effective two-level trapped ions interacting with a laser field is investigated. Also, the entanglement decay and sudden death of entanglement is observed for Werner state in common types of environmental noise [28].

4 Information Dynamics

Quantum information science has emerged as one of the most exciting scientific developments in the past decade. Let us assume that Alice and Bob have coded information, say a quantum secret key, in their shared entangled state. But due to the reality there is no isolated systems, the entangled state which carries the information interacts with its surroundings. These undesirable interactions cause a loss of information.

The aim of this section is investigating the effect of the thermal and squeezed reservoirs on the dynamics of information which is carried by the shared entangled state. In this treatment, two phenomena are investigated, the disturbance of information and the exchange information between the shared state and its local environments.

One says that a system is *disturbed* when its initial and final states do not coincide. Since the information is coded on the input states, then it may be quantified in terms of fidelities [29]. The closeness of the output quantum state ρ_f to the input one ρ_i is expressed by the quantum fidelity $\mathcal{F}$, where $0 \le \mathcal{F} \le 1$ and the disturbance, $\mathcal{D}$ is [30],

$$\mathcal{D} = 1 - \mathcal{F}, \quad \mathcal{F} = \mathrm{Tr}\{\rho_f \rho_i\}. \tag{13}$$

Figure 3(a), shows, the behavior of the disturbance of information in the presences of the thermal reservoir. It is clear that the disturbance increases as the scaled time, Γt increases. For small values of the thermal photons the disturbance increases gradually at the expense of the fidelity of the transformed state. For scaled time $\Gamma t \ge 4$, the disturbance, $\mathcal{D}$ reaches its maximum value. This means that the input and the output states are completely different and the entangled state converted to separable state (see Fig. 2(a)). As one increases the thermal

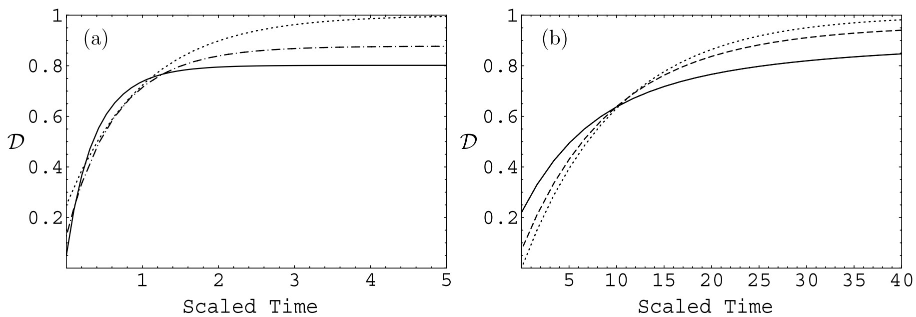

Fig. 3 The *dot*, *dash-dot* and the *solid curves* represent the Disturbance $\mathcal{D}$, for a system is initially prepared in maximum entangled state (**a**) for the thermal reservoir with $\mathcal{N}_1 = \mathcal{N}_2 = 0.00001, 0.2, 0.6$ (**b**) for the squeezed reservoir with $\mathcal{M}_1 = \mathcal{M}_2 = 0.001, 0.2, 0.4$ and $\mathcal{N}_1 = \mathcal{N}_2 = n = 0.2$

photon numbers, the disturbance increases with time and reaches to a constant value as soon as the entanglement disappears. So, the entangled state is completely separable and there is no more information to be disturbed. For the values which cause a sudden-death of entanglement as depicted in Fig. 1(a), the disturbance sudden becomes constant. This means that at these values there is a sudden death of information.

Figure 3(b), describes the behavior of the disturbance of information in the presence of the local squeezed reservoir. In general, the same behavior is seen as that depicted for the thermal reservoir, but the disturbance $\mathcal{D}$ increases slowly and consequently the time of information loss is large. This behavior is due to the long-lived entanglement as seen in Fig. 2(b). So, for small values of the squeezed reservoir parameters, the time of disturbed information can be infinite.

To quantify the exchange information between the state and the environment during the evaluation, we use the entropy exchange [31–33],

$$S_e = -\mathrm{Tr}\{\rho \log \rho\}. \tag{14}$$

If there is no interaction between the system and its surroundings, then the entropy exchange is zero and consequently there is no information loss from the system. One can look at the environment as an Eavesdropper, who wants to gain more information from the entangled system, which carries this information. In Fig. 4, we investigate the dynamics of this phenomenon in the presences of thermal and squeezed environments. In both cases, the exchange information increases as one increases the reservoir parameters, but as soon as the state turns into a separable state, the exchange information decreases. This means that there is no more interaction with the local environment, therefore the exchange information becomes a constant. The effect of the thermal reservoir is seen in Fig. 4(a), where for small values of the mean photon numbers $\mathcal{N}_1 = \mathcal{N}_2 = 10^{(-4)}$, the exchange entropy increases to reach its maximum values and then decreases gradually and becomes a constant at $\Gamma t \geq 5$. We notice that, at this time the state turns into a separable state (see Fig. 2(a)), so there is no more exchange between the environment and the state. As one increases the mean photon number, the exchange information becomes constant at $\Gamma t \cong 1.3$. For larger values of the mean photon numbers, say $\mathcal{N}_1 = \mathcal{N}_2 = 0.2$, the exchange information becomes constant much earlier. So, we can conclude that the behavior of the exchange information in the presence of the local squeezed reservoir is the same as that shown for the thermal reservoir. But the time in which the exchange information becomes constant is much larger than that depicted for the thermal case. Science in the squeezed reservoir case the entanglement is long-lived.

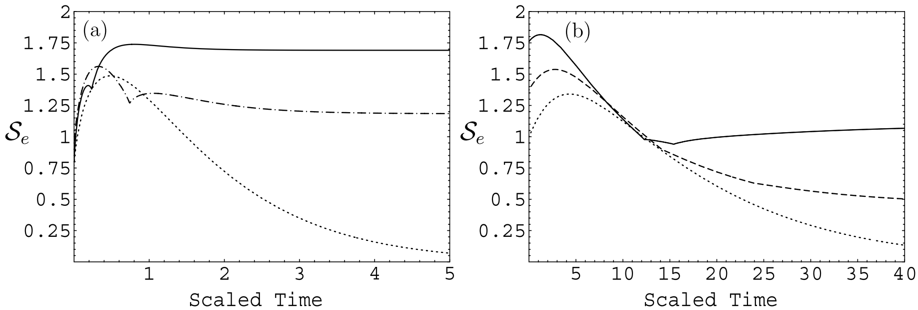

Fig. 4 The exchange information, (**a**) for the thermal reservoir with same values in as Fig. 3(a). (**b**) for the squeezed reservoir with same values as in Fig. 3(b)

Eventually, we show that the coded information in the traveling state could be subject to violation. Two strategies of violation have been considered, disturbance and exchange information. It is clear from these results, by controlling the environmental parameters, one can increases the time of lived entanglement and consequently reduces the violations.

5 Conclusion

In this contribution, the dynamics of entangled state passes through a thermal or squeezed reservoirs is investigated. The phenomenon of the entanglement decay and the sudden death of entanglement are shown for both reservoirs. We show that the entanglement lived longer for the squeezed reservoir. Also, the disturbance of information is discussed for both environment, where it is very sensitive to the thermal reservoir parameters. For large values of the thermal photon reservoir, the information is suddenly disturbed, but it is disturbed gradually for the squeezed reservoir. The loss of information is quantified by the means of the entropy exchange between the environment and the shared entangled state. For both environments, the exchange information increases and becomes a constant when the system turns into separable state. For thermal reservoir, the exchange information becomes constant faster than that for the squeezed reservoir.

One can look at the phenomena of disturbance and exchange information as a type of violation of information. So, by controlling the environmental parameters one can increases the robustness the state which carries the encrypted information and consequently reduces the lose of information. This study could be useful in building quantum computer.

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